Engineering Notes

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Sensitivity of Suboptimal Fixed-Range Flight Paths

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Nomenclature

C_{C0} = zero-lift drag coefficient
C_{Ci} = induced drag coefficient
D = drag
E = specific energy level
H = Hamiltonian
h = altitude
M = Mach Number
T = thrust
t = time
V = velocity
W = aircraft weight
W_f = fuel weight consumption
x = distance
λ = Euler-Lagrange variable
γ = flight-path angle
σ = specific fuel consumption

Subscripts

$(\ )_c$ = cruise
$(\ )_0$ = initial values
$(\ )_f$ = final values

Introduction

THE sensitivity of fuel performance along suboptimal minimum fuel trajectories is examined using a modified energy state approximation. The suboptimal path for nominal conditions will be determined and the sensitivity of fuel performance along the nominal path is investigated for variations in thrust, drag coefficients, aircraft weight, specific fuel consumption, and atmospheric conditions. Once the performance sensitivity along the nominal trajectory is established, the trajectory will be adjusted to be fuel optimal for each of the variations. The adjusted performance will then be compared with the performance along the nominal trajectory to determine if the path adjustment improves performance.

Analysis

Using specific energy, altitude, and downrange distance as state variables the point mass equations are easily developed.[1] The performance index is given by

$$J=\int_{t_0}^{t_f} \sigma T \, \mathrm{d}t \tag{1}$$

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using the Hamiltonian formulation, Schultz and Zagalsky[2] have shown that a fuel optimal trajectory results from the following

Maximum Thrust Energy Climb

$$\max_h \left[\frac{V(T-D)/W}{\sigma T - \sigma_c D_c V/V_c} \right] \tag{2}$$

Intermediate Thrust Cruise Point

$$\min_{h,v} \quad (\sigma D/V) \text{ with } T=D \tag{3}$$

Maximum Range Descent

$$\min_h \quad (D/W) \text{ with } T=0 \tag{4}$$

Note that Eq. (2) is similar to the optimizing functional for the minimum fuel problem with unspecified range.[1] The only difference being the additional term $(\sigma_c D_c/V_c)$ in the denominator which represents the fuel consumed per unit distance calculated at the cruise point. Equation (4) represents the maximum range glide with $T=0$. The total minimum fuel trajectory is a combination of arcs determined from Eqs. (2-4). The combination of arcs depends on whether or not the cruise point occurs at an energy level that is lower than the final energy level. Both Schultz and Zagalsky showed that the arc determined from Eqs. (2-4) satisfy the first-order necessary condition for fuel optimality. However, Speyer[3] has shown that these conditions do not satisfy the generalized Legendre-Clebsch condition for optimality. Specifically, Speyer demonstrated that a cruise point with intermediate thrust control is not fuel optimal. This result is true if thrust and flight-path angle are considered as controls. Schultz has recently shown that for an expanded model with thrust and lift as controls that the cruise arc is optimal.

Numerical Results

A more complete version of the numerical results is given in Ref. 5. The contours of the functional $(\sigma D/V)$ with its minimum value defining the optimal cruise point are shown in Fig. 1. The location of the cruise point is given by $M=0.80$

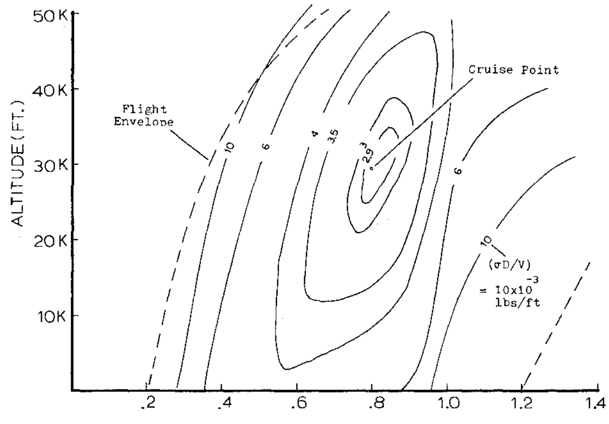

Fig. 1 Contours of the cruise function 4($\sigma D/V$) with the associated minimum value.

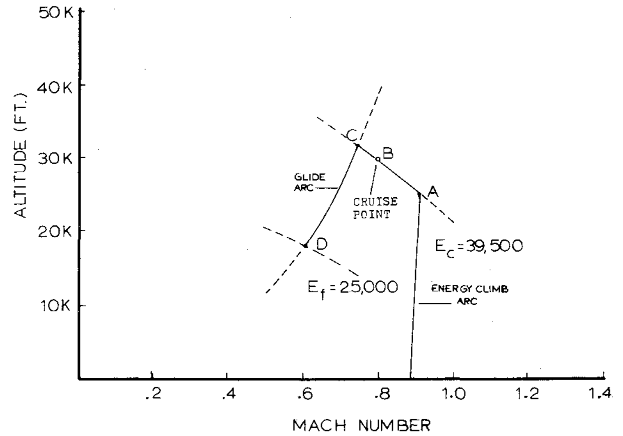

Fig. 2 Nominal minimum fuel-fixed range trajectory for the case $E_f < E_c$.

and $h = 29619$ ft. The value of the functional at this cruise point is

$$(\sigma_c D_c / V_c) = 2.58 \times 10^{-3} \text{ lb/ft} \tag{5}$$

and the cruise energy level is $E_c = 39{,}500$ ft. To determine the minimum fuel trajectory during an energy climb from E_0 to E_f, the functional value in Eq. (5) is used in Eq. (2). We will investigate the fuel performance sensitivity of the nominal minimum fuel trajectory for $E_f = 25{,}000$ ft ($E_f < E_c$) with $E_0 = 1{,}000$ ft. The trajectory $E_f < E_c$ is shown in Fig. 2. For $E_f < E_c$ there is a maximum thrust energy climb up to the cruise energy level followed by a zoom climb to the cruise point. After an intermediate thrust cruise ($T = D$), there is a constant energy zoom climb to the glide arc (point C) followed by a maximum range glide to the final energy level at point D.

Performance Sensitivity

We will now consider the performance sensitivity of fuel consumption to variations in aerodynamic parameters and atmospheric conditions along the nominal trajectory.

Nominal Conditions

The nominal atmospheric conditions are specified by the Standard Day atmosphere values[4] and the nominal aerodynamic values are those for an F-4 aircraft in a Standard Day atmosphere. Using Eqs. (2-4), the nominal trajectory for $E_f < E_c$ is given in Fig. 2. The nominal fuel consumption along this trajectory is 3618 lbs. This nominal value is used to determine relative sensitivity for parametric variations.

Parametric Variations

Variations in selected parameters along the nominal trajectory are used for performance sensitivity. Aerodynamic variations will be investigated for specific parameters while atmospheric variations will be investigated for the Hot Day atmosphere and the Cold Day atmosphere as specified by MIL-STD-210A. Using Eq. (1) with the expression for fuel consumption, $\dot{W}_f = \sigma T$, the expression for fuel consumption between specific energy levels is given by

$$\Delta W_f = \int_{E_I}^{E_2} \frac{\sigma W}{V(1 - D/T)} \, dE \tag{6}$$

Numerical results from Ref. 4 indicate that the fuel consumption during the energy climb is inversely proportional to thrust and directly proportional to specific fuel consumption, aircraft weight, and drag. However, the sensitivity of fuel consumption to atmospheric conditions is less clear. The sensitivities for specific parametric variations along the nominal trajectory are presented in Tables 1 and 2. From these results we see that fuel consumption is directly proportional to the drag terms, aircraft weight and specific fuel consumption; and inversely proportional to thrust. For example, increasing aircraft weight by 20% along the nominal trajectory causes a 19.5% increase in fuel consumption. The greatest per-

Table 1 Fuel consumption sensitivity of parameter variations $E_f < E_c$

Parameter	Variation	Fuel consumption (lb)	Sensitivity (%)
Nominal	None	3618	....
	+20%	3597	-0.6
Thrust	+10%	3636	-0.3
	-10%	3633	0.4
	-20	3652	0.9
	+20%	4076	12.7
C_{D0}	+10%	3885	7.4
	-10%	3552	-1.8
	-20	3384	-6.5
	+20%	4133	14.2
C_{Di}	+10%	3998	10.5
	-10%	3590	-.7
	-20	3462	4.3
	+20%	4463	23.4
Weight	+10%	4086	12.9
	-10%	3377	-6.7
	-20%	3062	-15.4
	+20%	4342	20.0
SFC	+10%	3980	10.0
	-10%	3256	-10.0
	-20%	2895	-20.0

Table 2 Fuel comsumption sensitivity to atmospheric variations for $E_f < E_c$

Atmosphere	Fuel consumption (1b)	Sensitivity (%)
Standard Day	3618	0
Hot Day	3686	1.9
Cold Day	3631	0.3

Table 3 Fuel consumption sensitivity for adjusted flight paths ($E_f \leq E_c$)

Parameter	Variation	Fuel consumption (1b)	Sensitivity (%)
Nominal	None	3618	0
	+20%	3596	-.6
Thrust	+10%	3636	-.3
	-10%	3633	.4
	-20%	3642	.9
	+20%	4070	12.5
C_{D0}	+10%	3878	7.2
	-10%	3553	1.8
	-20%	3394	-6.2
	+20%	4121	13.9
C_{Di}	+10%	3589	-.8
	-20%	3455	-4.5
	+20%	4414	22.0
Weight	+10%	3962	9.5
	-10%	3365	-7.0
	-20%	3035	-16.1
	+20%	4342	20.0
SFC	+10%	3980	10.0
	-10%	3256	-10.0
	-20%	2894	-20.0

formance sensitivity occurs for variations in aircraft weight and specific fuel consumption. However, fuel consumption appears to be relatively insensitive to variations in thrust and atmospheric variations.

Path Adjustment

Performance sensitivity may be compensated by adjusting the nominal trajectory to be fuel optimal for the specific variation under consideration. As a first step, optimal cruise points are determined for each variation using Eq. (3). Only variations in the drag coefficients, aircraft weight, and atmospheric conditions cause a shift in the cruise point. The cruise point shifts upward as the drag coefficients and aircraft weight increase, and it shifts downward as the parameters decrease; while a Hot Day atmosphere causes the cruise point to shift upward and a Cold Day atmosphere causes a downward shift. Using the values for $(\sigma_c D_c / V_c)$ calculated at these adjusted cruise points, the adjusted minimum fuel trajectory for each variation is determined. The fuel performance sensitivity for each variation is given in Table 3. Path adjustment causes very little, if any significant improvement in fuel consumption for any of the parametric or atmospheric variations. For example, when aircraft weight is increased by 20% along the nominal path, fuel consumption increases by 23%. However, when the nominal trajectory is adjusted to be fuel optimal for a 20% weight increase, the fuel consumption is increased by 22% representing a 1% improvement. Obviously, adjusting the nominal trajectory to be fuel optimal for the parameters examined here fails to provide significant improvement in fuel consumption.

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Real Model Following Control

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Introduction

THE technique of real model following control (RMF) has been shown to be amenable to the solution of many aircraft control problems. Commencing with the work of Kalman[1] and Tyler[2] extensive use has been made of linear optimal control theory in the design of RMF controllers. Available designs include the partial state feedback controller of Winsor and Roy[3] and the stability augmentation and mode decoupling controller of Yore.[4] While optimal control theory provides an extremely flexible synthesis technique a structural approach may, nevertheless, provide a superior design. In particular, it may be possible to achieve 'perfect' following, i.e. perfect matching of the dynamics of the compensated plant to those of the model, without rescourse to a high-gain controller. Further, an algebraic control law may be readily combined with a parameter estimation scheme to provide an adaptive capability.

Following the work of Erzberger[5] on the implicit model following problem various structural approaches have been adopted in synthesizing RMF controllers. Curran[6] exploited the concept of 'equicontrollability' to present an approximate design technique. An asymptotic RMF control law was derived by Chan[7] for the class of plants and models whose output vectors are identically their state vectors. Landau and Courtiol[8] have considered the adaptive model following problem for the same class of systems. Lowe,[9] using the second method of Lyapunov, derived an asymptotic control law applicable to single variable systems. In this paper a control law is obtained for a class of linear multivariable systems and the necessary and sufficient conditions for perfect following are derived. This control law reduces to that of Lowe for single variable systems and the derivation parallels the Lyapunov synthesis technique.

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Asymptotic Solution

Consider the linear time invariant multivariable plant

$$\dot{x}_p = A_p x_p + B_p u_p$$
$$y_p = C_p x_p \tag{1}$$

where x_p is an n state vector, u_p an m input vector and y_p an m output vector. A_p, B_p and C_p are matrices of appropriate dimensions. It is assumed that the triple (A_p, B_p, C_p) is controllable and observable. The prespecified linear time invariant model is described by

$$\dot{x}_m = A_m x_m + B_m u_m$$
$$y_m = C_m x_m \tag{2}$$

where the dimension of x_m, the state vector, is arbitrary, u_m is an m input vector and y_m is an m output vector.

The model following error e is defined as

$$e(t) = y_m - y_p \tag{3}$$

The design problem is to synthesize u_p such that $e(t) \to 0$ in the steady state. For a given plant and model the necessary and sufficient conditions for the existence of a perfect RMF control law must also be derived.

The time derivative of the error is given by

$$\dot{e} = \dot{y}_m - \dot{y}_p \tag{4a}$$
$$= C_m A_m x_m + C_m B_m u_m - C_p A_p x_p - C_p B_p u_p \tag{4b}$$

Let the input to the plant be decomposed as

$$u_p = K\, e + u_p^* \tag{5}$$

where K is a gain matrix to be determined.

Substituting Eq. (5) into Eq. (4b) yields

$$\dot{e} = -C_p B_p K e + C_m A_m x_m + C_m B_m u_m - C_p A_p x_p - C_p B_p u_p^* \tag{6}$$

It is required that

$$\lim_{t \to \infty} e(t) = 0 \tag{7}$$